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## WEAK TRANSITIONS OF HEAVY MESONS AND THE QUARK MODEL

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### Résumé

Nous soulignons les caractéristiques du modèle des quarks et ses succès dans la description du spectre et des transitions de hadrons lourds ou légers, comparant brièvement aux premiers principes de QCD. Pour montrer l'utilité actuelle des techniques et intuition physique du modèle, nous présentons un modèle des quarks semi-relativiste des facteurs de forme des mésons qui présente l'invariance d'échelle d'Isgur-Wise, et des corrections à celle-ci. Comme exemples d'applications, nous considérons les facteurs de forme semi-leptoniques et des désintégrations non leptoniques de mésons à saveur lourde. Nous posons la question de la violation de la factorisation, et montrons comment le modèle des quarks peut donner des indications sur ce problème.

### Abstract

We underline the general features of the constituent quark model and its success in the description of the spectrum and transitions of hadrons made up of light or heavy quarks, briefly compared to first principles of QCD. To show the present usefulness of the techniques and intuitive insight of the model we present a semirelativistic quark model of meson form

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factors that exhibits Isgur-Wise scaling and corrections to it. As examples of applications, we consider semileptonic form factors and non-leptonic decays of heavy flavor mesons. We address the issue of the violation of factorization, and show how the quark model can give hints on the problem.

## 1 Success and limits of the quark model

Following earlier remarks<sup>1</sup>, we will first briefly describe the main characteristics of the Constituent Quark Model (CQM), comparing it to the first principles of QCD, that we believe to be the right theory of strong interactions. We underline the relations and differences between both approaches, in particular which aspects of the CQM are understood in QCD, and also the successes of the CQM that still wait for rigorous explanations.

1) The CQM assumes a fixed number of constituents in a given hadron.

On the contrary, in QCD the number of constituents is not fixed, since  $\alpha_s(Q^2)$  grows with the distance, and in particular the number of gluons and quarks in the hadron wave function depends on  $Q^2$ .

2) In the CQM one considers constituents masses ; for the light quarks these are of the order  $m \simeq 0.3$  GeV (from  $\mu_p = \frac{1}{2m} = \frac{2.79}{2M_N}$ ). The constituent masses, together with a non-relativistic approximation, results in an approximate SU(6) symmetry for the light hadrons, non-strange and strange, that works reasonably well, mainly in the baryon sector.

In QCD, one starts with current masses in the Lagrangian, masses that for the lightest quarks  $u, d$  are of the order of a few MeV, resulting in an approximate Chiral Symmetry, that is dynamically broken. The constituent mass of the quark model can be identified with the quark mass dynamically generated by this spontaneous chiral symmetry breaking. However, the approximate SU(6) symmetry of the CQM, although empirically successful, does not have a theoretical basis. One must recall that attempts have been made to include the phenomenon of dynamical symmetry breaking in quark models, including dynamically generated mass, quasi-Goldstone mesons and degeneracy of the vacuum. These are introduced thanks to a second quantized chiral-symmetric version of the relativistic potential. But these attempts have not yet reached the stage of a complete phenomenology comparable to the CQM.

3) In the CQM one assumes a Schrödinger equation with a flavor-independent confining potential with a short distance and a long distance pieces :

$$V(r) = -\frac{\kappa}{r} + \lambda r + C \quad (1)$$

(for heavy quarkonia,  $\kappa = 0.5$ ,  $\lambda \simeq 0.2$  GeV<sup>2</sup>). Empirically, one finds that a Lorentz vector short distance, that in QCD would correspond to one-gluon exchange (OGE), and a Lorentz Scalar long distance pieces are favored, as regards spin-dependent forces. Indeed, within this hypothesis, one has  $v^2/c^2$  spin-dependent corrections to the spectrum, a short distance spin-spin interaction interaction,

$$V_{SS}(\mathbf{r}) = \frac{\alpha_1 \cdot \alpha_2}{6m_1 m_2} \Delta V_V(\mathbf{r}) \Rightarrow \frac{16\pi\alpha_s}{3} \frac{\alpha_1 \cdot \alpha_2}{6m_1 m_2} \delta(\mathbf{r}) \quad (\text{OGE}) \quad (2)$$

a  $\mathbf{L} \cdot \mathbf{S}$  force

$$V_{LS}(\mathbf{r}) = \frac{\mathbf{L} \cdot \mathbf{S}}{2m^2 r} \left( 3 \frac{dV_V}{dr} - \frac{dV_S}{dr} \right) \quad (m_1 = m_2) \quad (3)$$

and a tensor one

$$V_T(\mathbf{r}) = \frac{S_{12}}{12m_1m_2} \left( \frac{1}{r} \frac{dV_V}{dr} - \frac{d^2V_V}{dr^2} \right) . \quad (4)$$

The ratio of level differences in quarkonium would be, in the hypothesis of pure Lorentz vector potential (with  $\beta = < \frac{\lambda}{r} > / < \frac{\alpha_s}{r^3} >$ ) :

$$R = \frac{M(\chi_2) - M(\chi_1)}{M(\chi_1) - M(\chi_0)} = \frac{4}{5} \frac{8 + 7\beta}{8 + 4\beta} > \frac{4}{5} \quad (\text{Lorentz Vector}) \quad (5)$$

while, assuming the short distance potential to be Lorentz vector and the long distance one Lorentz scalar :

$$R = \frac{4}{5} \frac{16 - 5\beta}{16 - 2\beta} < \frac{4}{5} \quad (\text{SD Vector, LD Scalar}) . \quad (6)$$

Experiment gives for the  $c\bar{c}$  system  $R(\chi_c) = 0.48$  while for the  $b\bar{b}$  one  $R(\chi_b) = 0.64$ ,  $R(\chi'_b) = 0.58$ , strongly pointing to the spin-orbit force being like for a Lorentz scalar. Moreover, one has ( $M_{cog}(^3P_J)$ ,  $M(^1P_1)$ ) are independent of  $V_T$ ) :

$$M_{cog}(^3P_J) = M(^1P_1) \quad (7)$$

that is very well verified in charmonium (Table 2), where  $M_{cog}$  is the average mass of  $\chi_0$ ,  $\chi_1$ ,  $\chi_2$ . It is important to emphasize that there is no long range spin-spin force, as it would be if the long distance potential were a Lorentz vector.

These empirical hints of the quark model are well justified in QCD. Indeed, confinement is deduced from lattice QCD at strong coupling<sup>2</sup>. More realistic calculations on the lattice even with dynamical fermions suggest a potential of the form (1), as we can see in Fig. 1<sup>3</sup>. Moreover, Wilson loop calculations with an expansion in  $1/m_Q$  confirm the Schrödinger picture with the potential (1) and the form of the  $\mathbf{S} \cdot \mathbf{S}$  and  $\mathbf{L} \cdot \mathbf{S}$  interactions as suggested by the quark model<sup>4</sup>.

4) In the CQM the vacuum is trivial. In QCD the vacuum is highly non-trivial. One has three main sets of phenomena : (i) Non perturbative effects connected with confinement:  $< GG >$  condensate. (ii) Dynamical breaking of chiral symmetry,  $< q\bar{q} >$  condensate, quasi Nambu-Goldstone bosons  $\pi$  and  $K$ . (iii) Non-perturbative effects in the U(1) sector :  $< G\tilde{G} >$  condensate, non-triviality of the vacuum due to instantons, special status of the  $\eta'$  pseudoscalar meson, possibility of CP violation in the strong interactions. Concerning (ii), as pointed out above, one can formulate generalizations of the quark model with spontaneous chiral symmetry breaking<sup>5</sup>.

5) The Quark Model is a non-relativistic model, the expansion parameter being  $v/c$ . The mean value of this parameter is quite different in the different bound systems :

Table I  
Quark mass, level spacing  $\omega$  and internal  $v^2/c^2$  for the different quarkonia

Quarkonia	$m_q$ (GeV)	$\omega$ (GeV)	$v^2/c^2$
$b\bar{b}$	5.12	0.48	0.13
$c\bar{c}$	1.82	0.46	0.27
$q\bar{q}$	0.3	0.5-0.7	0.7
$Q\bar{q}$	0.3	0.46	0.7

For the light quarks the internal quark motion is relativistic. In the quark model one considers two types of relativistic effects : (i) Relativistic corrections due to the binding, for example in current matrix elements like the nucleon axial coupling

$$\frac{G_A}{G_V} = \frac{5}{3} (1 - 2\delta) \quad (8)$$

where  $\delta = O\left(\frac{v^2}{c^2}\right)$ , has the right sign and order of magnitude (there are also  $O(\alpha_s)$  radiative corrections). (ii) Relativistic effects due to the center-of-mass motion, that come out in hadron form factors for example, as we will see below.

In QCD, there are three important limiting regimes around which one can formulate very fruitful systematic expansions in small parameters: (i) The hard or large momentum regime, in which the expansion parameter is  $\alpha_s(Q^2) \sim 1/\log(Q^2/\Lambda^2)$ . (ii) Chiral symmetry limit for light quarks where the expansion parameters are, grosso modo,  $m_q/\Lambda$  and  $p_\mu/\Lambda$  (small momenta). Chiral symmetry translates into effective light hadron chiral Lagrangians. (iii) Heavy Quark limit where the expansion parameter is essentially  $\Lambda/m_Q$ . This leads to the useful Heavy Quark Symmetry.

As we will see below, the relativistic effects due to the center-of-mass motion in the CQM make the link with the Heavy Quark Effective Theory of QCD.

6) The CQM gives a good overall description of spectra and transitions of light and heavy hadrons, for the ground state and the excitations. We show a few examples in Fig. 2 (energy levels of first excited light baryons), Table 2 (energy levels of heavy quarkonia  $c\bar{c}$ ,  $b\bar{b}$ ), Table 3 (radiative transitions of baryon isobars) and Table 4 (E1 transitions in the  $b\bar{b}$  system). However, one must say that for the quasi-Goldstone bosons, mainly the pion, and the U(1) sector, the quark model finds difficulties.

The quark model provides an intuitive understanding of the phenomena in terms of bound states and its wave functions, that is used almost unconsciously by everybody working on phenomena involving hadrons.

The rigorous methods of QCD, like lattice QCD, confirm many results of the quark model for the ground state hadrons, and in addition can treat the pion as a Goldstone boson. However, these methods cannot yet give an overall view of the spectra and transitions of hadrons as the CQM does, especially because it handles very easily the excited states, which are hardly accessible to the fundamental methods. Moreover, the success of the non-relativistic quark model for light quarks, even if it is possibly amended by relativistic corrections, is not understood in terms of QCD.

Table 2

Quarkonia  $c\bar{c}$ ,  $b\bar{b}$  levels (*NR* : non-relativistic model ;  $\mathbf{S} \cdot \mathbf{S}$  : hyperfine splitting) ; \* : input

$c\bar{c}$	Exp. (GeV)	$NR[6]\mathbf{S} \cdot \mathbf{S}[7]$	$b\bar{b}$	Exp. (GeV)	$NR[6]\mathbf{S} \cdot \mathbf{S}[7]$
$\eta_c \ 1^1S_0$	2.980	2.956	$\eta_b \ 1^1S_0$		9.30
$J/\psi \ 1^3S_1$	3.097	3.095*	$\Upsilon \ 1^3S_1$	9.460	9.46*
$\eta_c \ 2^1S_0$	3.594	3.550	$\eta_b \ 2^1S_0$		9.93
$\psi \ 2^3S_1$	3.686	3.684*	$\Upsilon \ 2^3S_1$	10.023	10.05
$\psi \ 3^3S_1$	4.040	4.110	$\Upsilon \ 3^3S_1$	10.355	10.40
$\psi \ 4^3S_1$	4.415	4.460	$\Upsilon \ 4^3S_1$	10.580	10.67
			$\eta_b \ 4^1S_0$		10.57
			$\Upsilon \ 5^3S_1$	10.865	10.92
			$\eta_b \ 5^1S_0$		10.82
			$\Upsilon \ 6^3S_1$	11.019	
			$\eta_b \ 6^1S_0$		11.04
$\chi_c \ 1^3P_J$ (c.o.g.)	3.525	3.522*	$\chi_b \ 1^3P_J$ (c.o.g.)	9.900	9.96
$h_c \ 1^1P_1$	3.526		$h_b \ 1^1P_1$		9.96
$\psi \ 1^3D_1$	3.770	3.810	$\Upsilon \ 1^3D_1$		10.20
			$\chi_b \ 2^3P_J$ (c.o.g.)	10.261	10.31
			$h_b \ 2^1P_1$		10.31
$\psi \ 2^3D_1$	4.159	4.190	$\Upsilon \ 2^3D_1$		10.50

Table 3  
Radiative transitions  $N^* \rightarrow N\gamma$  (data from PDG 1994).

	$A_{3/2}^p$ th.	$A_{3/2}^p$ exp.	$A_{1/2}^p$ th.	$A_{1/2}^p$ exp.	$A_{3/2}^n$ th.	$A_{3/2}^n$ exp.	$A_{1/2}^n$ th.	$A_{1/2}^n$ exp.
<b>56,0<sup>+</sup></b> $P_{33}$ (1236)	- 178	- 257	- 103	- 141				
<b>70,1<sup>-</sup></b> $S_{11}$ (1535)			160	68			- 109	- 59
<b>70,1<sup>-</sup></b> $D_{13}$ (1520)	112	163	- 29	- 22	- 112	- 137	- 30	- 62
<b>70,1<sup>-</sup></b> $D_{15}$ (1670)	0	18	0	18	- 53	- 70	- 38	- 50
<b>70,1<sup>-</sup></b> $S_{31}$ (1650)			47	30				
<b>70,1<sup>-</sup></b> $D_{33}$ (1670)	91	91	92	114				
<b>56,2<sup>+</sup></b> $F_{15}$ (1688)	70	135	- 15	- 14	0	- 35	41	27

Table 4  
E1 radiative transitions in the  $b\bar{b}$  system.  
The ratio  $\Gamma/k^3 \propto (2J+1)$ , predicted by the Non-relativistic Quark Model, is very well satisfied by the data.

$\Upsilon(2S) \rightarrow$ $\chi_b(1^3P_J) \gamma$	$\Gamma(keV)$ exp.	$\Gamma(keV)$ th. <sup>8</sup>	$\Upsilon(3S) \rightarrow$ $\chi_b(2^3P_J) \gamma$	$\Gamma(keV)$ exp.	$\Gamma(keV)$ th. <sup>8</sup>
$J = 0$	1.9	1.43	$J = 0$	1.5	1.55
$J = 1$	2.95	2.27	$J = 1$	2.8	2.5
$J = 2$	2.90	2.24	$J = 2$	2.7	2.75

## 2 Weak transitions of heavy mesons

Weak decays of mesons are governed by the Cabibbo-Kobayashi-Maskawa (CKM) matrix :

$$(\bar{u}, \bar{c}, \bar{t})\gamma_\mu(1 - \gamma_5) \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (9)$$

that, in the Wolfenstein parametrization and expansion in powers of the Cabibbo angle  $\lambda = V_{us} = \sin \theta_C$ , writes :

$$V \cong \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} . \quad (10)$$

In this expansion, unitarity is approximate,  $A$  is of order  $O(1)$ , and the complex number  $\rho - i\eta = e^{-i\delta}$  is responsible of the CP violation, with  $\rho$  and  $\eta$  of order  $O(1)$ .

The different decays depend on different CKM matrix elements. For example, the semileptonic decays  $B \rightarrow D(D^*)\ell\nu$ ,  $B \rightarrow \pi(\rho)\ell\nu$ ,  $D \rightarrow K(K^*)\ell\nu$ ,  $K \rightarrow \pi\ell\nu$  depend respectively on  $V_{cb}$ ,  $V_{ub}$ ,  $V_{cs}$  and  $V_{us}$ .

## 2.1 Heavy Quark Symmetry

Assume a  $Q\bar{q}$  system made of a heavy quark  $Q$  and a light antiquark  $\bar{q}$ . As we have seen above, in the Quark Model, the system is described by a Hamiltonian with a spin independent potential plus spin dependent terms that are inversely proportional to the quark masses :

$$H = \frac{p^2}{2\mu} + V(r) + \frac{16\pi\alpha_s}{3} \frac{\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2}{6m_Q m} \delta(\mathbf{r}) + \dots \quad \mu = \frac{m_Q m}{m_Q + m} . \quad (11)$$

In the infinitely heavy quark limit, for  $m_Q \rightarrow \infty$ ,  $H \rightarrow$  finite limit : the reduced mass  $\mu \rightarrow m$ , the wave functions, binding energies, become independent of the flavor and spin of the heavy quark. The dynamics depends only then on the light quark degrees of freedom :  $H(\mathbf{r}, \mathbf{p}, \boldsymbol{\sigma}_2, m)$ . This has a number of interesting consequences for hyperfine and fine splittings :

$$\begin{aligned} \mathbf{S} \cdot \mathbf{S} \text{ splittings : } & M_{B^*} = M_B \quad M_{D^*} = M_D \\ \mathbf{L} \cdot \mathbf{S} \text{ splittings : } & M_{B^{**}} - M_B = M_{D^{**}} - M_D \end{aligned} \quad (12)$$

that are qualitatively satisfied by experiment, with small corrections to the infinitely heavy quark limit.

One can generalize to QCD these simple considerations, with an exact treatment of light degrees of freedom, gluons and light quarks<sup>9</sup>. For an arbitrary four-velocity of the heavy hadron  $v^\mu = P^\mu/M$  ( $v^2 = 1$ ), one can write the four-momentum of the heavy quark in the form

$$p_Q^\mu = m_Q v^\mu + k^\mu \quad (13)$$

where  $k$  is a typical residual momentum due to the binding within the heavy-light hadron wave function. Then, the heavy quark propagator can be approximated by

$$\frac{m_Q \not{v} + \not{k} + m_Q}{(m_Q v + k)^2 - m_Q^2} \cong \frac{\not{v} + 1}{2v \cdot k} . \quad (14)$$

The interesting feature of this expression is that the heavy propagator is independent of the heavy mass and depends only on the hadron four-velocity and a residual momentum  $k$ , very small compared to  $m_Q v$ .

One has a generalization of an atomic picture. Like in the hydrogen atom, if the hadron is at rest, the heavy quark acts as a static source of color (up to  $1/m_Q$  corrections). However, here the picture is fully relativistic, since it is generalized to any hadron four-velocity, and this will have interesting consequences for the hadron form factors. One has an independence of the spin and flavor of the heavy quark, leading to a symmetry  $[SU(2N_f)]_v$ , the subindex  $v$  meaning that there is a symmetry for any given  $v^\mu$ .

This Heavy Quark Symmetry, discovered by Isgur and Wise<sup>9</sup>, has important consequences for the semileptonic form factors in the heavy-to-heavy case (like  $B \rightarrow D(D^*)\ell\nu$ ) and also, although weaker results, for the heavy-to-light case (like  $B \rightarrow \pi(\rho)\ell\nu$ )<sup>10</sup>.

Let us define the form factors :

$$\begin{aligned}
\langle P_i | V_\mu | P_j \rangle &= \left( p_\mu^i + p_\mu^j - \frac{M_j^2 - M_i^2}{q^2} q_\mu \right) f_+(q^2) + \frac{M_j^2 - M_i^2}{q^2} q_\mu f_0(q^2) \\
\langle V_i | A_\mu | P_j \rangle &= (M_i + M_j) A_1(q^2) \left( \varepsilon_\mu^* - \frac{\varepsilon^* \cdot q}{q^2} q_\mu \right) - \\
&\quad - A_2(q^2) \frac{\varepsilon^* \cdot q}{M_i + M_j} \left( p_\mu^i + p_\mu^j - \frac{M_j^2 - M_i^2}{q^2} q_\mu \right) + 2M_i A_0(q^2) \frac{\varepsilon^* \cdot q}{q^2} q_\mu \\
\langle V_i | V_\mu | P_j \rangle &= i \frac{2V(q^2)}{M_i + M_j} \varepsilon_{\mu\nu\rho\sigma} p_j^\nu p_i^\rho \varepsilon^{*\sigma} \quad .
\end{aligned} \tag{15}$$

The Heavy Quark Symmetry implies scaling laws for the heavy-to-light form factors<sup>10</sup>, that apply for example to the form factors  $B \rightarrow \pi(\rho)\ell\nu$  and also to  $B \rightarrow K(K^*)$ , that could be related to the non-leptonic decays  $B \rightarrow K(K^*)\psi$  within the factorization approximation. The scaling law applies at fixed three-momentum  $\mathbf{q}$ , small compared to the heavy quark mass, i.e. four momentum transfer close to its maximum value :

$$\frac{1}{\sqrt{m_Q}} \langle K(K^*), \mathbf{q} | J^\mu | P_Q \rangle \cong Cte. \quad (|\mathbf{q}|, M_K, M_{K^*} \ll m_Q) \quad . \tag{16}$$

This implies, for the different form factors :

$$\begin{aligned}
f_0(\mathbf{q}), A_1(\mathbf{q}) &\sim \frac{1}{\sqrt{m_Q}} \\
f_+(\mathbf{q}), V(\mathbf{q}), A_0(\mathbf{q}), A_2(\mathbf{q}) &\sim \sqrt{m_Q} \quad (|\mathbf{q}| \ll M_Q, q^2 \cong q_{max}^2) \quad .
\end{aligned} \tag{17}$$

In the heavy-to-heavy case, like in  $B \rightarrow D(D^*)\ell\nu$ , stronger relations are valid for any value of  $q^2$  (up to corrections in  $1/m_Q$ , in practice  $1/m_c$ )<sup>9</sup> :

$$\begin{aligned}
\frac{\sqrt{4M_B M_{D^*}}}{(M_B + M_{D^*})} A_0(q^2) &= \frac{\sqrt{4M_B M_{D^*}}}{(M_B + M_{D^*})} A_2(q^2) = \frac{\sqrt{4M_B M_{D^*}}}{(M_B + M_{D^*})} \frac{A_1(q^2)}{\left[1 - \frac{q^2}{(M_B + M_{D^*})^2}\right]} = \\
\frac{(\sqrt{4M_B M_D})}{(M_B + M_D)} f_+(q^2) &= \frac{\sqrt{4M_B M_D}}{(M_B + M_D)} \frac{f_0(q^2)}{\left[1 - \frac{q^2}{(M_B + M_D)^2}\right]} = \sqrt{\frac{4M_B M_{D^*}}{(M_B + M_{D^*})}} V(q^2) = \xi(v \cdot v')
\end{aligned} \tag{18}$$

where  $\xi(v \cdot v')$  is the so-called Isgur-Wise function that depends only on the product of the initial and final four-velocities. It satisfies the normalization  $\xi(1) = 1$  (i.e. at  $\mathbf{q} = 0$  or  $q^2 = q_{max}^2$ ).

These relations and the normalization are very important because they allow an almost model-independent determination of the CKM matrix element  $V_{cb}$  from the decay  $B \rightarrow D^*\ell\nu$ .

The differential decay rate  $B \rightarrow D^*\ell\nu$  writes

$$\frac{d\Gamma}{dw} = \frac{G^2}{48\pi^3} F(M_{D^*}, M_B, w) |V_{cb}|^2 \eta_A^2 \widehat{\xi}^2(w) \tag{19}$$



where  $w$  is the variable  $w = v_{D^*} \cdot v_B = \frac{M_B^2 + M_{D^*}^2 - q^2}{2M_B M_{D^*}}$ ,  $F(M_{D^*}, M_B, w)$  is a known function,  $\eta_A$  is a known short distance QCD correction, and the function  $\hat{\xi}(w)$  differs from the exact heavy quark limit function  $\xi(w)$  by  $O(1/m_Q)$  corrections. In particular, at the normalization point, there are quadratic corrections<sup>11</sup> :  $\hat{\xi}(1) = 1 + \delta_{1/m^2}$ . From a linear fit  $\hat{\xi}(w) = 1 - \hat{\rho}^2(w - 1)$  one finds (CLEO experiment<sup>12</sup>)

$$|V_{cb}| = 0.038 \pm 0.006 \pm 0.004 \quad . \quad (20)$$

## 2.2 Quark model of semileptonic form factors

This is a weak binding model that takes into account the relativistic center-of-mass motion of the hadrons<sup>13</sup>, as referred to in 5) of section 1. The model provides an intuitive link between the Quark Model and the Heavy Quark Symmetry.

The total hadron wave function writes

$$\Psi_{\mathbf{P}}^{tot}(\{\mathbf{p}_i\}) = \delta(\sum_i \mathbf{p}_i - \mathbf{P}) \Psi_{\mathbf{P}}(\{\mathbf{p}_i\}) \quad (21)$$

with the internal wave function

$$\Psi_{\mathbf{P}}(\{\mathbf{p}_i\}) = N \left[ \prod_i S_i(\mathbf{P}) \right] \Psi_{\mathbf{P}=0}(\{\tilde{\mathbf{p}}_i\}) \quad . \quad (22)$$

The model accounts for two important relativistic effects :

1) Lorentz contractions of the wave function

$$\begin{aligned} \tilde{\mathbf{p}}_{iT} &= \mathbf{p}_{iT} \\ \tilde{p}_{iz} &= \frac{E}{M} p_{iz} - \frac{P}{M} \varepsilon_i = \sqrt{1 - \beta^2} \left( p_{iz} - \frac{P}{M} m_i \right) \\ \tilde{\varepsilon}_i &= \frac{E}{M} \varepsilon_i - \frac{P}{M} p_{iz} \cong m_i \end{aligned} \quad (23)$$

2) Lorentz boost of Dirac spinors

$$S_i(\mathbf{P}) = \sqrt{\frac{E + M}{2M}} \left( 1 + \frac{\mathbf{f}_i \cdot \mathbf{P}}{E + M} \right) \quad . \quad (24)$$

The calculation of current matrix elements is relatively simple in the equal velocity frame (EVF), where the initial and final hadrons have the same modulus of the velocity  $\beta$ , and opposite in direction. An interesting relation in this frame is :

$$1 - \beta^2 = \frac{4M_i M_f}{(M_i + M_f)^2} \frac{1}{\left[ 1 - \frac{q^2}{(M_i + M_f)^2} \right]} \quad . \quad (25)$$

The form factors read, in this model :

$$f_+(q^2) = \frac{\sqrt{4M_i M_f}}{(M_i + M_f)} \frac{1}{\left[ 1 - \frac{q^2}{(M_i + M_f)^2} \right]} I(q^2) (1 + X_+)$$

$$\begin{aligned}
V(q^2) &= \frac{\sqrt{4M_i M_f}}{(M_i + M_f)} \frac{1}{\left[1 - \frac{q^2}{(M_i + M_f)^2}\right]} I(q^2)(1 + X_V) \\
A_1(q^2) &= \frac{\sqrt{4M_i M_f}}{M_i + M_f} I(q^2)(1 + X_1) \\
A_2(q^2) &= \frac{\sqrt{4M_i M_f}}{(M_i + M_f)} \frac{1}{\left[1 - \frac{q^2}{(M_i + M_f)^2}\right]} I(q^2)(1 + X_2)
\end{aligned} \tag{26}$$

etc. In these expressions  $I(q^2)$  is the wave function overlap, and  $X_+$ ,  $X_V$ ,  $X_1$ ,  $X_2$  ( $M_i$ ,  $M_f$ ,  $q^2$ )  $\sim O(m)$  are corrections proportional to the spectator quark mass.

The model has a number of interesting features. It satisfies heavy-to-light Isgur-Wise scaling (16), (17), and also heavy-to-heavy Isgur-Wise scaling (18) with the Isgur-Wise function

$$\xi(v \cdot v') = \frac{2}{1 + v \cdot v'} I(v \cdot v') \quad \xi(1) = 1 \tag{27}$$

where  $I(v \cdot v')$  depends on the overlap of the wave functions at rest :

$$I(v \cdot v') \cong \int \Phi_{\mathbf{P}_f=0}^+ \left( \mathbf{p} + \frac{m}{M_i + M_f} \tilde{\mathbf{q}} \right) \Phi_{\mathbf{P}_i=0} \left( \mathbf{p} - \frac{m}{M_i + M_f} \tilde{\mathbf{q}} \right) d\mathbf{p} \tag{28}$$

where  $\tilde{\mathbf{q}}$  is the Lorentz contracted momentum transfer

$$\tilde{\mathbf{q}}^2 = (1 - \beta^2) \mathbf{q}^2 = (M_f + M_i)^2 \frac{[(M_f - M_i)^2 - q^2]}{[(M_f + M_i)^2 - q^2]} = (M_i + M_f)^2 \left( \frac{v \cdot v' - 1}{v \cdot v' + 1} \right) \quad . \tag{29}$$

The normalization  $\xi(1) = 1$  results then from the normalization of the wave function at rest. In the harmonic oscillator model one has :

$$\xi(v \cdot v') = \frac{2}{1 + v \cdot v'} \exp \left[ -\frac{m^2 R^2}{\sqrt{2}} \left( \frac{v \cdot v' - 1}{v \cdot v' + 1} \right) \right] \tag{30}$$

an expression found elsewhere on different grounds<sup>14</sup>. The slope is given by

$$\rho^2 = -\xi'(1) = \frac{1}{2} + \frac{m^2 R^2}{2\sqrt{2}} \tag{31}$$

where the second term is the dominant one in the non-relativistic limit, and the first is a relativistic correction due to the boost of spinors.

Moreover, the model exhibits scaling corrections due to the spectator quark mass that point to a softening of the scaling laws (17) at fixed  $\mathbf{q}$ . These softening of the scaling is confirmed by lattice calculations within very large errors<sup>15</sup>. Also, neglecting hyperfine splitting, the model gives relations among the form factors that express the fact that, in the model, the total quark spin of the final meson equals the meson spin :

$$A_0(q^2) = f_+(q^2) \tag{32}$$

$$2M_f (M_i - M_f) f_0(q^2) = (M_i^2 - M_f^2 - q^2) A_1(q^2) - \frac{\lambda(M_i^2, M_f^2, q^2)}{(M_i + M_f)^2} A_2(q^2) \quad .$$

### 2.3 Example of phenomenological application

Let us consider the relation between the semileptonic form factors  $D \rightarrow K(K^*)\ell\nu$  and the form factors extracted from the non-leptonic decays  $B \rightarrow \psi K(K^*)$ . We have the following theoretical constraints and experimental data :

- 1) Heavy-to-light Isgur-Wise scaling near  $q^2 \cong q_{max}^2$ .
- 2) Data on  $D \rightarrow K(K^*)\ell\nu$  near  $q^2 \cong 0^{16}$  :

$$\begin{aligned} f_+^{sc}(0) &= 0.77 \pm 0.04 & V^{sc}(0) &= 1.16 \pm 0.16 \\ A_1^{sc}(0) &= 0.61 \pm 0.05 & A_2^{sc}(0) &= 0.45 \pm 0.09 \end{aligned} \quad (33)$$

- 3) Data on the decays  $B \rightarrow \Psi K(K^*)$ , that give information on the form factors at  $q^2 = m_\psi^2$  ( $L$  stands for longitudinal polarization) :

$$R = \frac{\Gamma(\bar{B}_d^0 \rightarrow \psi K^{*0})}{\Gamma(\bar{B}_d^0 \rightarrow \psi K^0)} \quad R_L = \frac{\Gamma_L(\bar{B}_d^0 \rightarrow \psi K^{*0})}{\Gamma_{tot}(\bar{B}_d^0 \rightarrow \psi K^{*0})} \quad (34)$$

Table 5

	$R$	$R_L$
Experiment	$1.64 \pm 0.34$ (CLEO) <sup>23</sup>	$0.66 \pm 0.10$ $^{+0.10}_{-0.08}$ (CDF) <sup>21</sup> $> 0.78$ (ARGUS) <sup>22</sup> $0.80 \pm 0.08 \pm 0.05$ (CLEO) <sup>22</sup>
BSWI <sup>17</sup>	4.23	0.57
BSWII <sup>18</sup>	1.61	0.36
GISW <sup>19</sup>	1.71	0.06
QCDSR <sup>20</sup>	7.60	0.36

Within the factorization hypothesis<sup>17</sup> these rates can be related to the  $B \rightarrow K(K^*)$  form factors, like for example :

$$A(\bar{B}_d^0 \rightarrow \psi K) = \frac{G}{\sqrt{2}} V_{cb} V_{cs}^* 2f_\psi m_B f_+(m_\psi^2) a_2 p \quad (35)$$

Table 6

	$R$	$R_L$	$\frac{f_+^{sc}(0)}{A_1^{sc}(0)}$	$\frac{V^{sc}(0)}{A_1^{sc}(0)}$	$\frac{A_2^{sc}(0)}{A_1^{sc}(0)}$	$\chi^2/DoF$
Exp.	$1.64 \pm 0.34$	$0.66 \pm 0.14$	$1.26 \pm 0.12$	$1.90 \pm 0.25$	$0.74 \pm 0.15$	
Fit	2.15	0.45	1.45	1.62	0.81	4.2

A simultaneous fit to  $D$  semileptonic form factors and to  $B$  non-leptonic decays is not very good (Table 6), but some trends of our model seems to be favored, namely :

- 1) Softened scaling :  $\frac{f_+^{sb}(q_{max}^2)}{f_+^{sc}(q_{max}^2)} = \left(\frac{m_D}{m_B}\right)^{\frac{1}{2}} \left(\frac{m_B+m_K}{m_D+m_K}\right)$ , etc.
- 2) Pole for the ratios  $f_+(q^2)/A_1(q^2)$ ,  $V(q^2)/A_1(q^2)$ ,  $A_2(q^2)/A_1(q^2)$ .
- 3) Weak  $q^2$  dependence of  $A_1(q^2)$ .

## 2.4 The factorization problem

In the precedent application, we have made use of factorization to establish a connection between non-leptonic decays and semi-leptonic form factors. For  $B$  mesons, factorization seems approximately correct at least for the so-called class I decays, that are of leading order at large  $N_c$ , as shown by CLEO results<sup>3</sup>. But it could happen that for the decays  $B \rightarrow \psi K(K^*)$ , that are subleading in  $1/N_c$ , violations of factorization (i.e. the recipe (35)) could modify the ratio  $R$  and even  $R_L$ . We can wonder whether this assumption has theoretical justification. First, we recall that this factorization principle is implemented in the standard approach of SVZ<sup>24</sup>, with particular values of  $a_1$  and  $a_2$ , which are found to be

$$a_1 = c_1 + \frac{c_2}{N_c} \quad , \quad a_2 = c_2 + \frac{c_1}{N_c} \quad (36)$$

where  $c_1, c_2$  are the coefficients in the effective weak Hamiltonian :

$$H = c_1(\bar{c}b)_1(\bar{s}c)_1 + c_2(\bar{c}c)_1(\bar{s}b)_1 \quad . \quad (37)$$

In fact it can be shown that this standard factorization is exact in the  $N_c \rightarrow \infty$  limit. However, as shown by a simple argument due to Shifman and by us<sup>25</sup>, the predicted rates cannot be correct at subleading order  $1/N_c$ , because they contradict duality. The argument is as follows. Let us consider as an example the Cabibbo suppressed process  $b \rightarrow c\bar{u}s$ . There are two types of decays corresponding to different color topologies, namely  $B_d \rightarrow D^+ D_s^-$  (class I) and  $B_d \rightarrow \psi K^0$  (class II). Let us consider the effective Hamiltonian (the subindex means color singlet). The decay rates into a pair of hadrons corresponding to classes I and II result, from factorization :

$$\begin{aligned} |A(B_s \rightarrow (\bar{s}c)_1(\bar{c}s)_1)|^2 &\sim \left[ c_1 + \frac{c_2}{N_c} \right]^2 \\ |A(B_s \rightarrow (\bar{c}c)_1(\bar{s}s)_1)|^2 &\sim \left[ c_2 + \frac{c_1}{N_c} \right]^2 \quad . \end{aligned} \quad (38)$$

Total quark decay rate differs from the sum of the expressions (38), since it is proportional to :

$$|A(b \rightarrow c\bar{u}s)|^2 \sim c_1^2 + c_2^2 + 2\frac{c_1 c_2}{N_c} \neq \left[ c_1 + \frac{c_2}{N_c} \right]^2 + \left[ c_2 + \frac{c_1}{N_c} \right]^2 \quad . \quad (39)$$

On the contrary, duality tells us that by summing over all mesons one should obtain total decay rate  $\propto c_1^2 + c_2^2 + 2c_1 c_2/N_c$ . Therefore, Shifman proposes a recipe: simply to impose duality by multiplying the subleading terms in the decay rates to hadrons by a factor  $x = 1/2$

$$\begin{aligned} |A(B_d \rightarrow (\bar{s}c)_1(\bar{c}s)_1)|^2 &\sim \left[ c_1 + x \frac{c_2}{N_c} \right]^2 \\ |A(B_d \rightarrow (\bar{c}c)_1(\bar{s}s)_1)|^2 &\sim \left[ c_2 + x \frac{c_1}{N_c} \right]^2 \quad . \end{aligned} \quad (40)$$

In our opinion, the reason for the discrepancy observed in (39) is that the color configurations  $(\bar{c}b)_1(\bar{s}c)_1$  and  $(\bar{c}c)_1(\bar{s}b)_1$  are not orthogonal, the overlap being of order  $1/N_c$  :

$$(\bar{c}b)_1(\bar{s}c)_1 = \frac{1}{N_c}(\bar{c}c)_1(\bar{s}b)_1 + \frac{\sqrt{N_c^2 - 1}}{N_c}[(\bar{c}c)_8(\bar{s}b)_8]_1 \quad . \quad (41)$$

We have considered a model with non-relativistic scalar quarks bound to color singlets by a color harmonic oscillator potential. This model takes into account of a Final State Interaction

that automatically restores duality or the conservation of probability. Indeed, summing over all mesons in the limit in which the radius  $R \rightarrow \infty$  one should find the free quark result. This is indeed the case, since one finds :

$$\begin{aligned} |A(B_d \rightarrow (\bar{s}c)_1(\bar{c}s)_1)|^2 &\sim \left[ c_1 + y \frac{c_2}{N_c} \right]^2 \\ |A(B_d \rightarrow (\bar{c}c)_1(\bar{s}s)_1)|^2 &\sim \left[ c_2 + (1-y) \frac{c_1}{N_c} \right]^2 \end{aligned} \quad (42)$$

with  $y(m_c, m_s)$  depending non-trivially on the masses, unlike the universal factor 1/2 proposed by Shifman. It is seen that FSI restores automatically duality. Notice by the way that expression (42), for an arbitrary value of  $y$ , is more general than Shifman's Ansatz and would as well restore duality. Another study of restoration of duality in the Quark Model approach may be found in [26]. The ratios  $R$  and  $R_L$  could be modified by this type of FSI effects, since by introducing spin there is no reason a priori that the ratio  $R_L$  would not be modified also.

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